Cellular Automata and the Game of Life

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**Abstract – Background: Conway’s Game of Life, or similar implementations that can be accessed through applications such as Golly, is a deceptively simple ‘game’ that is based on the concept of cellular automata.**

I. BACKGROUND

Cellular automata owes much of its origin to John von Neumann’s studies theorizing the creation of a self-replicating machine, which he referred to as an automaton. He envisioned this process would begin with a universal constructor that once given instructions can construct any other machine. He envisioned machine A floating in a reservoir of liquid containing building components required to build additional parts, part B which would create copies of the instructions furnished to A, and part C called the ‘control mechanism’ which would execute the given instructions. Von Neumann’s colleague at the Los Alamos National Laboratory, Stanislaw Ulam, suggested implementing his ideas using a 2D lattice. This was the groundwork of the concept now known as cellular automata. [1]

The concept was mainly one of academic interest until Cambridge mathematician John Conway developed his “Game of Life” which was popularized through Scientific American and tasked users with developing indefinite systems using its framework. Scientific American further lauded the game’s potential to open a new field of research in cellular automata. [2]

In 2002 Stephen Wolfram (of Wolfram Alpha computational knowledge engine) released his book *A New Kind of Science* which is a study of computational mathematics. In his reflections on his work he still marvels at the complexities that can be exhibited by even the simplest programs, and how the states in Game of Life relate to his Principle of Computational Equivalence as a computation that transforms input to output. [3]

II. IMPLEMENTATION

*A. Von Neumann’s Rules*

Von Neumann and Stanislaw Ulam set up the framework for which cellular automata is still based: a system of finite state automata whose state is determined by its position to adjacent finite state automata in an otherwise homogenous system laid out on an infinite array of cells. Each cell has a neighborhood which Neumann defined as the four orthogonal cells that shared its boundaries. In Von Neumann’s guidelines each cell would have a ‘relatively small’ number of 29 states. These states would achieve logical operations, transmission, wire branching, and construction/destruction of cells. [4] Using this arrangement von Neumann was able to outline a 200,000-cell system capable of self-reproduction, which he called a universal constructor. Von Neumann’s system was complicated compared to modern cellular automata models because he believed this level of complexity would be necessary for self-reproduction and hoped later to convert the results from the 2D model back to differential equations. [5]

*B. John Conway’s Rules*

While Conway kept the two-dimensional infinite and homogenous grid from Von Neumann he reduced the number of states to just two: alive or dead. He also increased the neighborhood of cells that could influence a cell’s evolution to eight, meaning that a cell diagonally adjacent was now in a cell’s neighborhood. He chose the name due to the similarity between the game and the growth and decline of living organisms. Conway used three guidelines in designing the rules: that no simple initial pattern should allow for limitless growth, that more complex initial patterns could grow without limit, and that the three end criteria were to fade away, settle into an unmoving final state, or settle into an oscillating state. He then established the genetic rules that governed play: a counter with two or three neighbors lived to the next evolution, a counter with fewer than two neighbors died from isolation, a counter with more than three counters died from overpopulation, and empty cells with exactly three adjacent live cells ‘birthed’ a new live cell. [6]

The game involves an initial state set by the user at t = 0 and for each increment of t all rules happen at the same time, with each new stage of the system referred to as an evolution. Lifeforms were cataloged, some of the simplest categories of which being still life which are stable unchanging patterns, oscillators which are patterns that change shape but return to their original position after several ticks, and gliders/spaceships which that translates across the matrix one set of cells at a time. Conway conjectured that no pattern could grow limitlessly and offered $50 to anybody who could disprove him, a reward that was claimed by Bill Gosper who developed a pattern that could generate a glider every 30 seconds called the Gosper glider gun. [2], [6]

*C. Stephen Wolfram’s Rules*

Many of the ‘rules’ derived by Stephen Wolfram in his text *A New Kind of Science* use elementary cellular automation in which each cell has two possible states (0 or 1) which is influenced in each evolutionary step by the three cells on top of it. As such each rule can be demonstrated by a table with several ‘T’ shapes, with the top of the T giving the state of the current evolution’s cells and the bottom of the T giving the state for the cell in the next evolution. Due to this most of Wolfram’s rules appear as pyramids, growing from a single cell downwards and outwards. [7]

III. ANALYSIS

*A. Relation to Finite Automata*

Cellular systems can be thought of as finite state machines, with each cell representing a state and each transition representing an arc providing the input from neighboring cells. Based on this they can also be expressed with regular languages and regular expressions. For instance, {(0|1)…} corresponds to all sequences of 0’s and 1’s. [8]

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